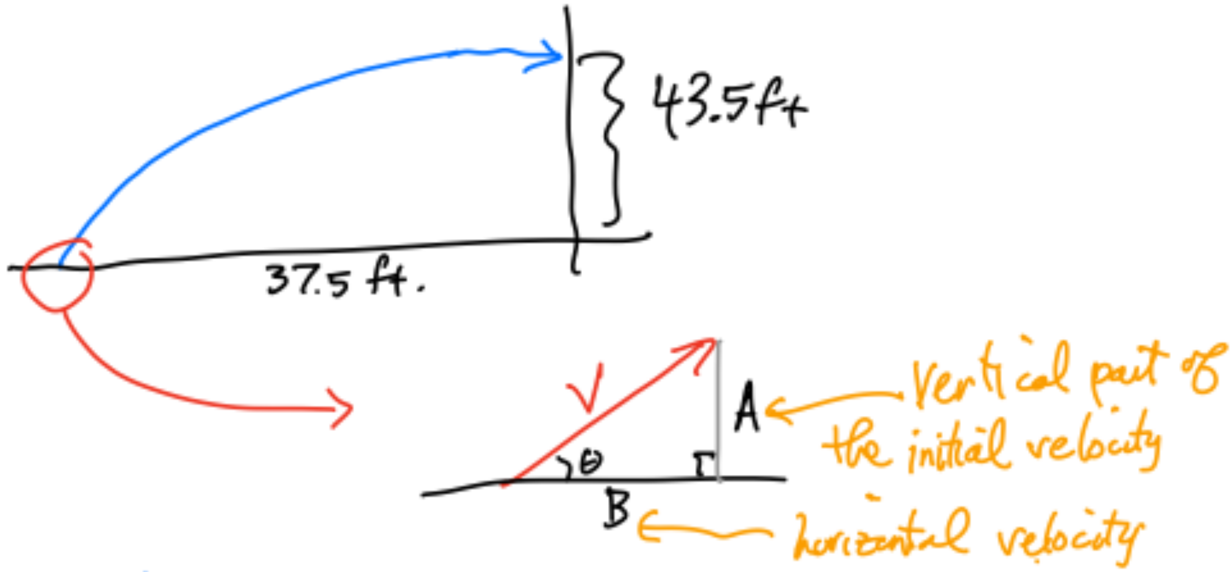


Vertial part:

$$s''(t) = -32.174 \text{ ft/s}^2$$

vertical velocity $s'(t) = -32.174t + (\text{Constant})$

$(\text{constant}) = A$ since $s'(0) =$ initial vertical velocity

$$\Rightarrow s'(t) = -32.174t + A$$

$\Rightarrow s(t) = -16.087t^2 + At + (\text{another constant})$ vertical position

when $t=0$, $s(t)=0 \Rightarrow$ (another constant) $= 0$.

$$\Rightarrow s(t) = -16.087t^2 + At$$

$$s(t) = -16.087t^2 + At$$
$$s'(t) = -32.174t + A$$

If t = time when the student arrives at the door.

$$s(t) = -16.087t^2 + At = 43.5$$
$$s'(t) = -32.174t + A = 0$$

$$A = 32.174t, \text{ or } t = \frac{A}{32.174}$$

$$\Rightarrow s(t) = -16.087\left(\frac{A}{32.174}\right)^2 + A\left(\frac{A}{32.174}\right) = 43.5$$

$$= A^2\Big[\underbrace{\left(\frac{-16.087}{32.174^2}\right) + \frac{1}{32.174}}_{\text{blah}}\Big] = 43.5$$

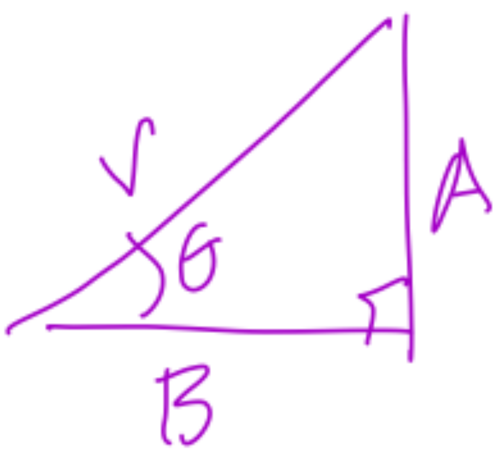
$$\Rightarrow A^2 = \frac{43.5}{\text{blah}} \Rightarrow A = \sqrt{\frac{43.5}{\text{blah}}}$$

$$A = 52.9 \text{ ft/s}$$

$$t_{\text{at top}} = \frac{A}{32.174} = 1.644 \text{ s}$$

$$Bt = 37.5$$

$$B = \frac{37.5}{1.644} = 22.8 \text{ ft/s}$$



$$V = \sqrt{A^2 + B^2}$$

$$\tan\theta = \frac{A}{B}$$

$$\theta = \arctan\left(\frac{A}{B}\right) \ldots$$

Question: Give an example of functions f & g such that $f(g(x)) = x$. ← f is a left inverse of g!

What is the relation between their derivatives?

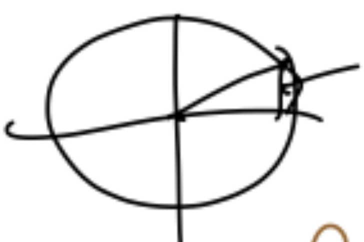
Examples: $f(x) = x^2$, $g(x) = \sqrt{x}$ || $f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$

yep.

$f(x) = \arcsin(x)$, $g(x) = \sin(x)$ || $f(g(x)) = \arcsin(\sin(x)) = x$. sometimes

↑ principal angle between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$.

If $x = \frac{\pi}{6}$, $\arcsin(\sin(x)) = \arcsin(\sin(\frac{\pi}{6})) = \arcsin(\frac{1}{2}) = \frac{\pi}{6}$. ✓



But, if $x = \pi$, $\arcsin(\sin \pi) = \arcsin(0) = 0$.

However if $f(x) = \sin(x)$
$g(x) = \arcsin(x)$,
then $f(g(x)) = \sin(\arcsin(x))$
$= x$. always.

sin is a left inverse of arcsin.

$f(x) = \frac{1}{2}x$

$g(x) = 2x$

$f(g(x)) = f(2x) = \frac{1}{2}(2x) = x$. ✓

other way also works.

Another one: $f(x) = \frac{1}{x}$

$g(x) = \frac{1}{x}$

$f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x$

✓

If f is a <u>left inverse</u> of g then $g'(x) = \dfrac{1}{f'(g(x))}$.

<u>Proof.</u> Given $f(g(x)) = x$, we differentiate both sides to get

$$f'(g(x)) \cdot g'(x) = 1$$

$$\Rightarrow \quad g'(x) = \frac{1}{f'(g(x))}. \quad \blacksquare$$

<u>Example:</u>

$$\left.\begin{aligned} e^{\ln(x)} &= x \\ \ln(e^x) &= x \end{aligned}\right\} \; e^x \text{ \& } \ln(x). \text{ are inverses!}$$

$$\Rightarrow \left(e^{\ln x}\right)' = (x)'$$

$$e^{\ln x} \cdot (\ln(x))' = 1$$

$$(\ln(x))' = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

$$\boxed{(\ln(x))' = \frac{1}{x}}.$$

Example:

$$(\ln(2x))' = \ln'(2x) \cdot 2$$

$$= \frac{1}{2x} \cdot 2 = \frac{1}{x}.$$

Why is this the same derivative as $(\ln(x))'$?

$$(\ln(2x))' = (\ln(2) + \ln(x))'$$
$$= (\ln(x))' = \frac{1}{x}. \checkmark$$

whew.

Also $(\ln(-x))' = \frac{1}{-x} \cdot (-1) = \frac{1}{x}. \checkmark$

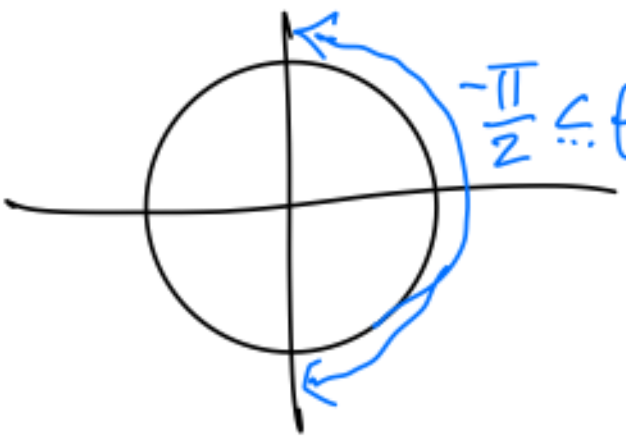
if x is negative

Inverse Trig fcns.

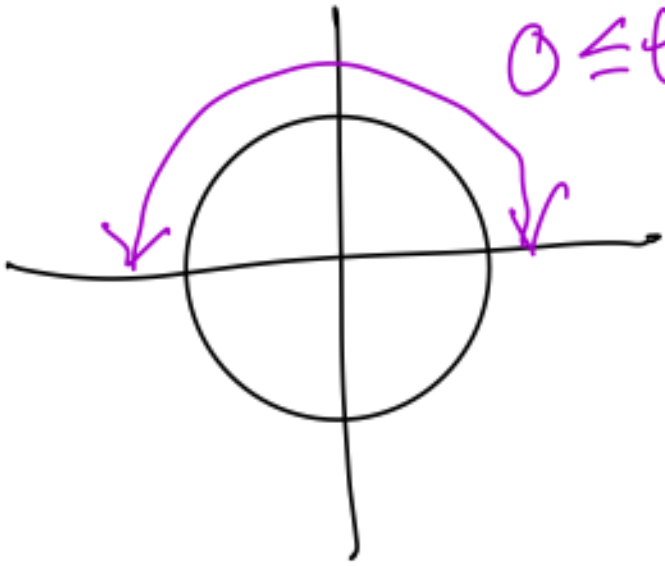
Interlude: Trig formulas.

arcsin(x) = principle angle whose sine is x.

arcsec(x) = principle angle whose secant is x.



$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ principal range of angles for arcsin, arctan, arc csc



$0 \le \theta \le \pi$ principle range of angles for arccos, arccot, arcsec

Simplifying procedure for expressions like

$\sin(\arctan(t))$

or $\sec(\arccos(x))$

Example: Simplify $\sin(\arctan(t))$

① let $\theta = \arctan(t)$

② Draw picture.

③ Find $\sin(\theta)$.

$\tan\theta = t = \frac{t}{1} \Rightarrow \sin(\theta) = \frac{t}{\sqrt{1+t^2}}$

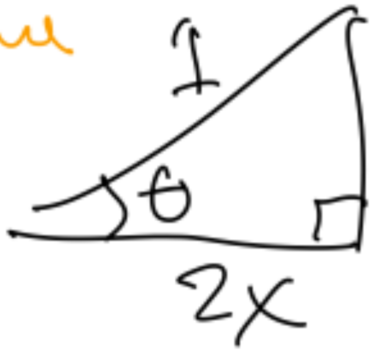
$$\therefore \sin(\arctan(t)) = \frac{t}{\sqrt{1+t^2}}$$

Example: Simplify $\tan(\arccos(2x))$

① $\theta = \arccos(2x)$

$$\cos\theta = 2x = \frac{2x}{1} = \frac{A}{H}$$

② Picture



$$\sqrt{1-(2x)^2} = \sqrt{1-4x^2}$$

③ $$\tan\theta = \frac{\sqrt{1-4x^2}}{2x}$$

$$\therefore \tan(\arccos(2x)) = \frac{\sqrt{1-4x^2}}{2x}$$

Problem: Find

$$(\arcsin(x))' = ?$$

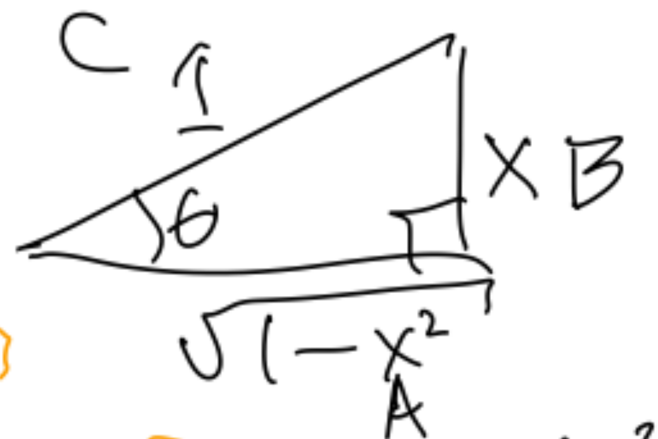
Pf: $\sin(\arcsin(x)) = x$

Derivative: $\cos(\arcsin(x)) \cdot (\arcsin(x))' = 1$

$$\Rightarrow (\arcsin(x))' = \frac{1}{\cos(\arcsin(x))}$$

$\theta = \arcsin(x)$

$\sin\theta = x$

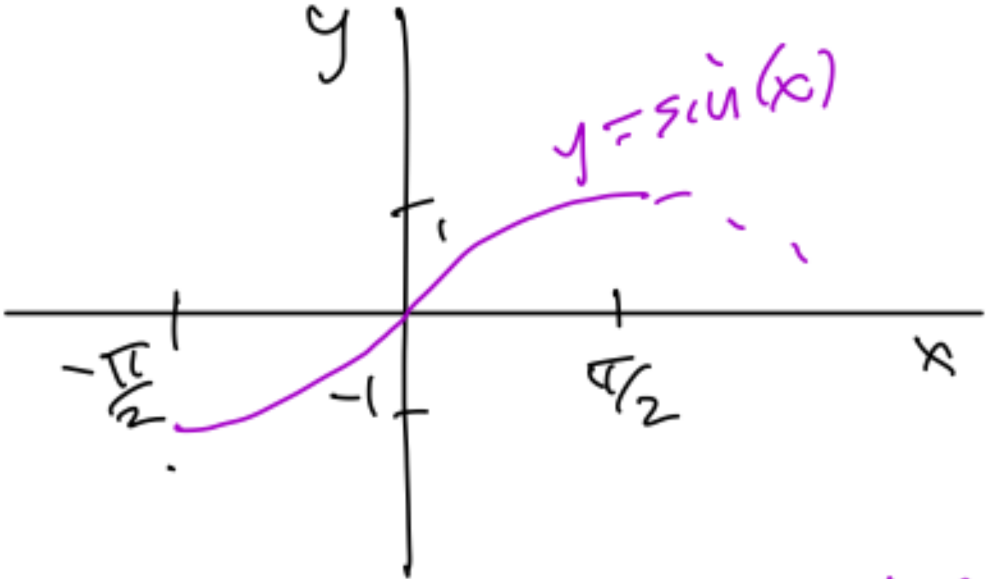


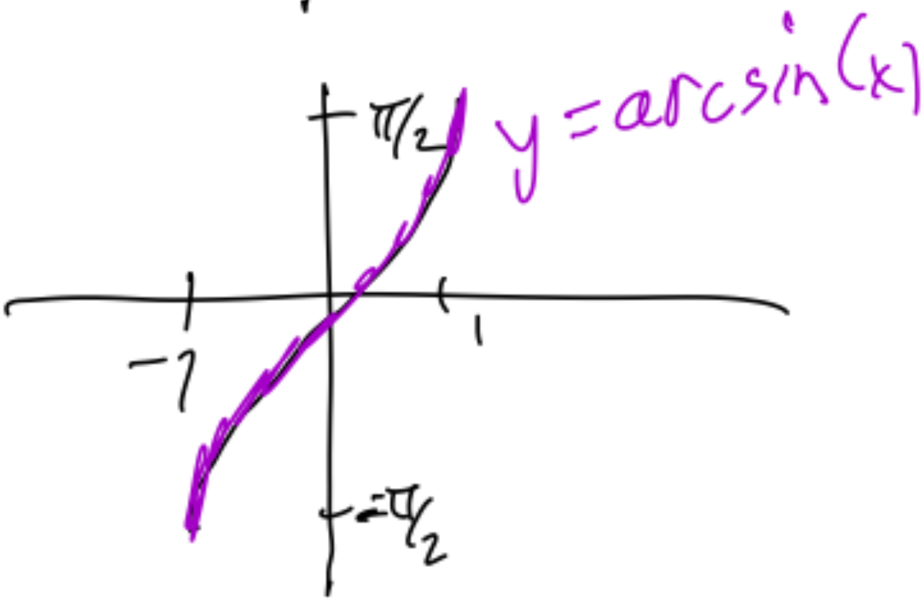
$\cos\theta = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

$A^2 + B^2 = C^2$

$A^2 = C^2 - B^2$

$$\boxed{(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}}$$





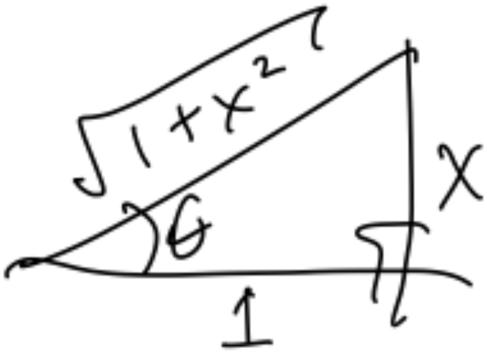
Example Derive the formula for $(\arctan(x))'$.

Proof: $\tan(\arctan(x)) = x$.

We differentiate:

$$\sec^2(\arctan(x)) \cdot (\arctan(x))' = 1.$$

$$(\arctan(x))' = \frac{1}{\sec^2(\arctan(x))}$$



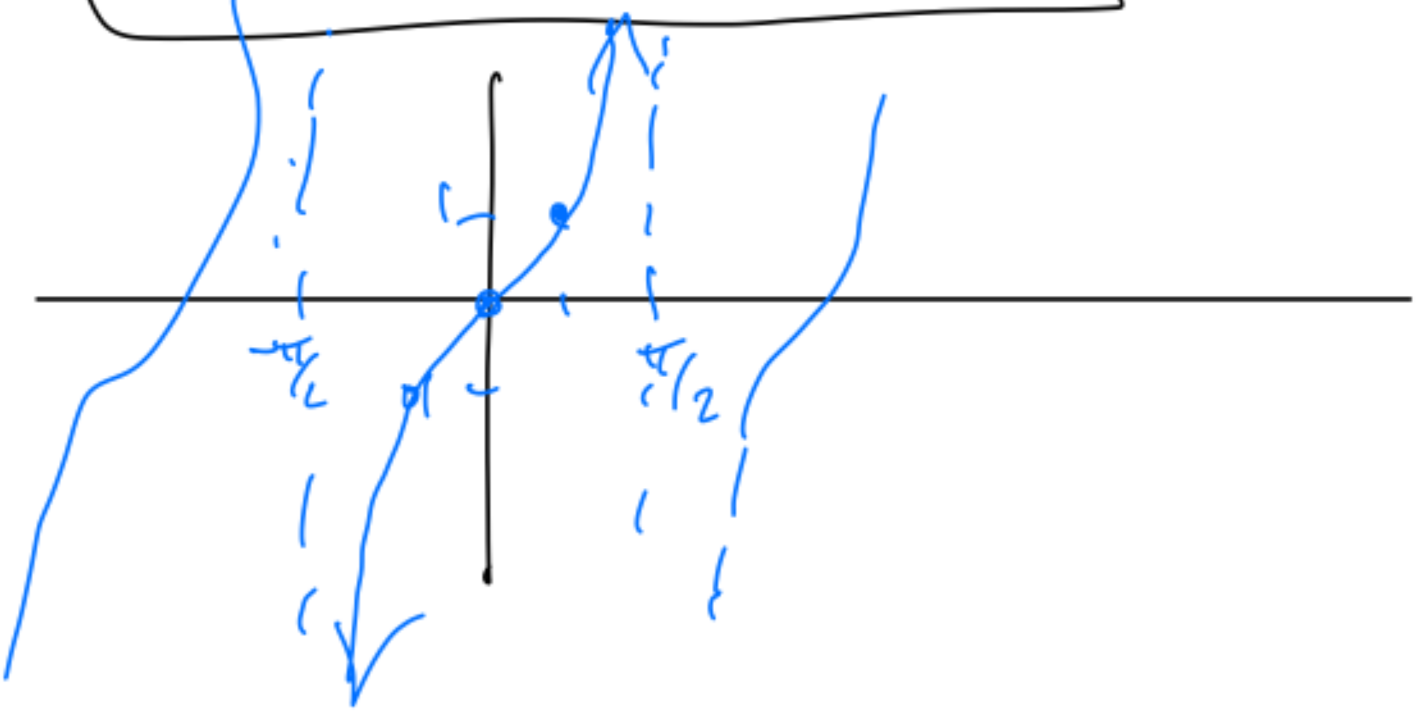
$$\theta = \arctan(x)$$

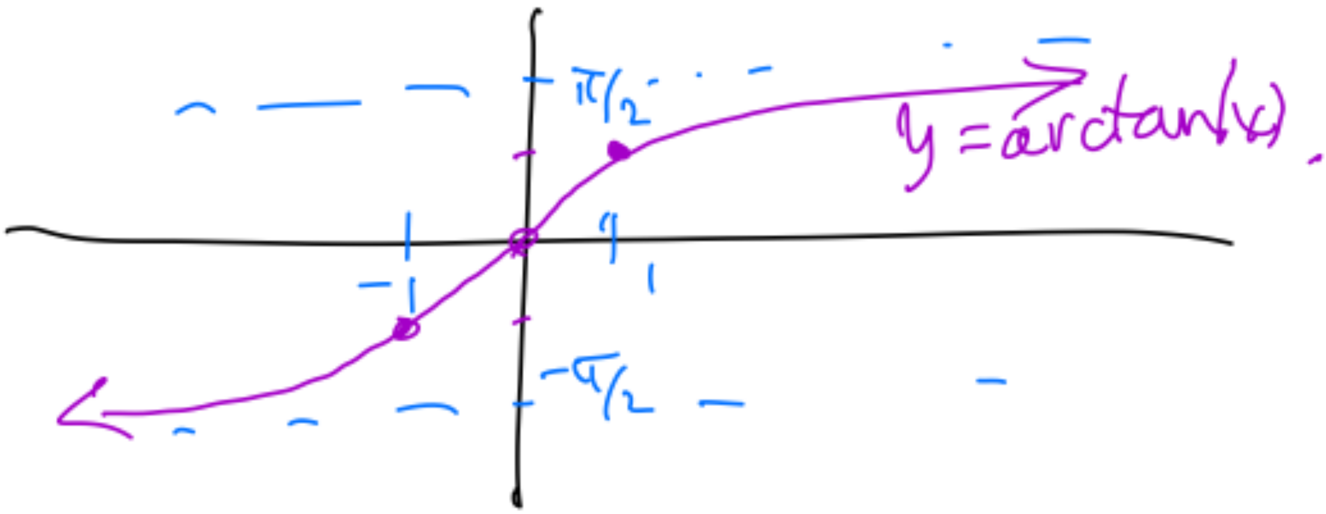
$$\tan(\theta) = x = \frac{x}{1}$$

$$\sec(\theta) = \sqrt{1+x^2}$$

$$\sec^2(\theta) = 1+x^2$$

$$\boxed{(\arctan(x))' = \frac{1}{1+x^2}}$$





(Example) Prove the formula for

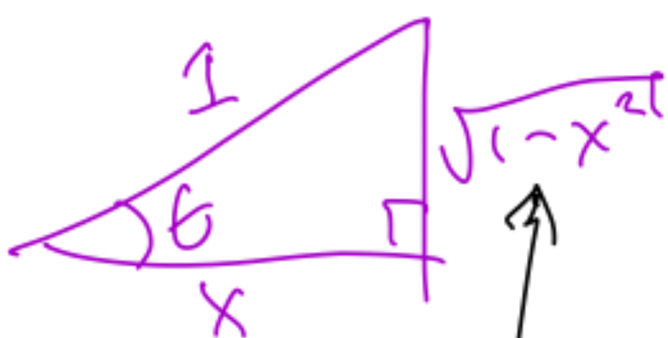
$$(\arccos(x))'$$

Pf. $\cos(\arccos(x)) = x$

Differentiation

$$-\sin(\arccos(x)) \cdot (\arccos(x))' = 1$$

$$\Rightarrow (\arccos(x))' = \frac{1}{-\sin(\arccos(x))}$$



$\theta = \arccos(x)$

$\cos\theta = x = \frac{x}{1}$

$x^2 + \text{Bubba}^2 = 1$

$\text{Bubba}^2 = 1 - x^2 \Rightarrow \text{Bubba} = \sqrt{1-x^2}$

$$\sin(\theta) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

$$\Rightarrow \boxed{(\arccos(x))' = \frac{1}{-\sqrt{1-x^2}}}$$

Note $(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$

$$\arccos(x) = \frac{\pi}{2} - \arcsin(x)$$
$$\operatorname{arccot}(x) = \frac{\pi}{2} - \arctan(x)$$
$$\operatorname{arccsc}(x) = \frac{\pi}{2} - \operatorname{arcsec}(x).$$

$$\Rightarrow (\operatorname{arccot}(x))' = -(\arctan(x))'$$
$$= -\frac{1}{1+x^2}.$$

Example Find $(\operatorname{arcsec}(x))'$.

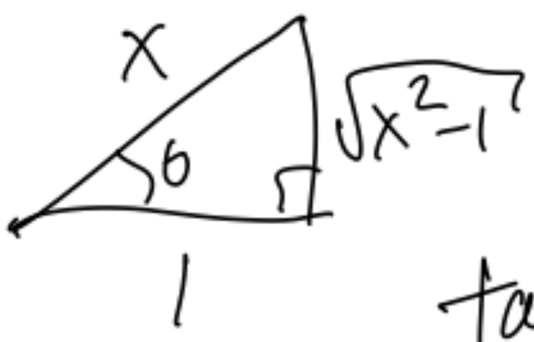
Pf: $\sec(\operatorname{arcsec}(x)) = x$

$$\underbrace{\sec(\operatorname{arcsec}(x))}_{x} \cdot \tan(\operatorname{arcsec}(x)) \cdot (\operatorname{arcsec}(x))' = 1.$$

$$\Rightarrow (\operatorname{arcsec}(x))' = \frac{1}{x\tan(\operatorname{arcsec}(x))}$$

$$\theta = \operatorname{arcsec}(x)$$

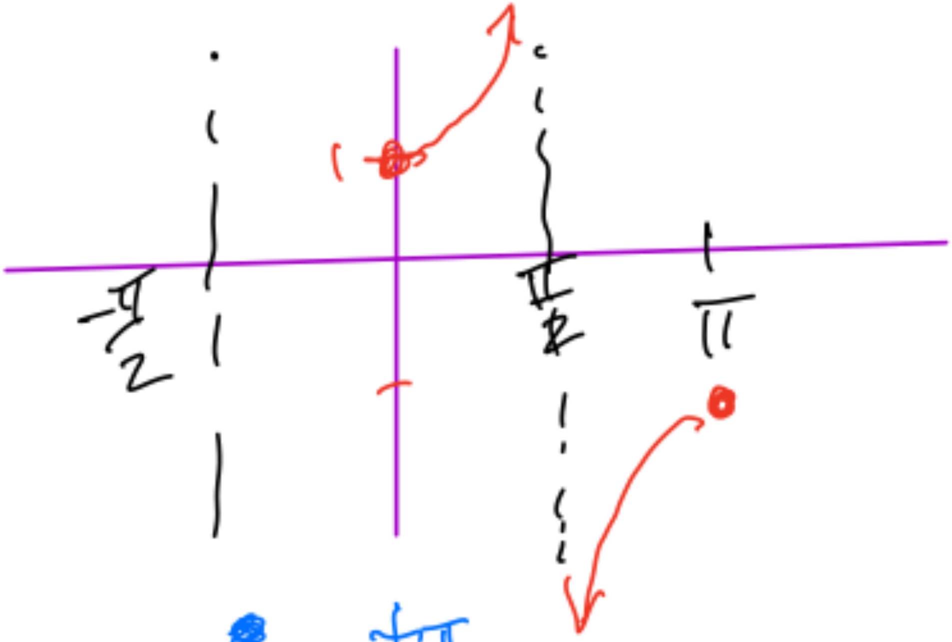
$$\sec\theta = x = \frac{x}{1}$$

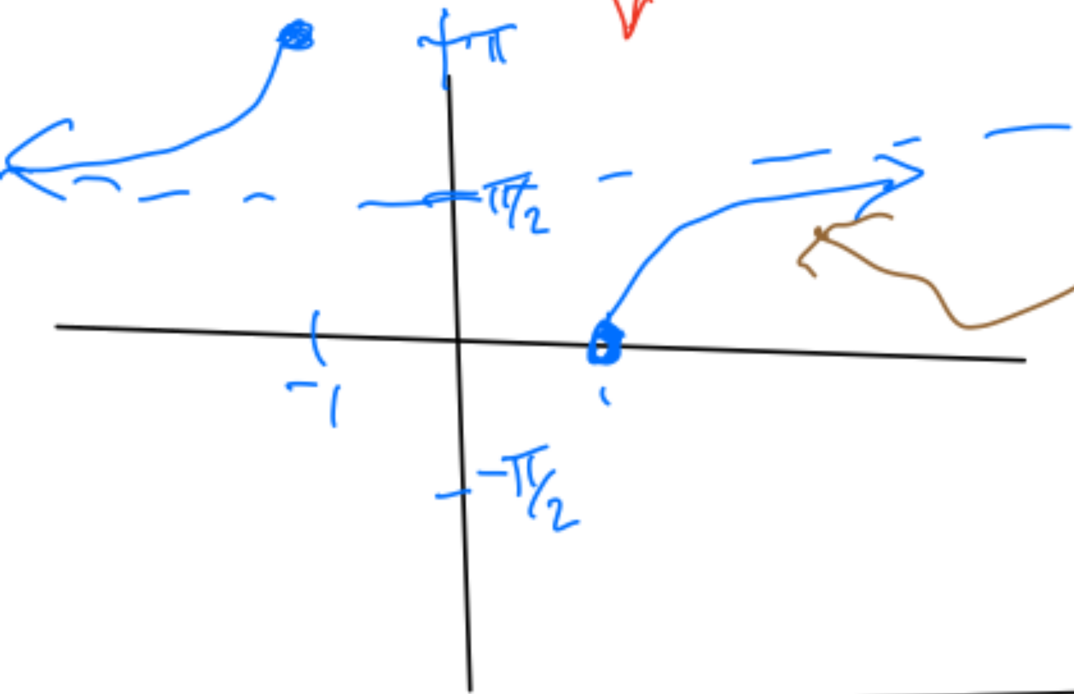


$$\tan\theta = \sqrt{x^2-1}$$

$$\Rightarrow (\operatorname{arcsec}(x))' = \frac{1}{x\sqrt{x^2-1}}$$

if x is positive





$$\Rightarrow \boxed{(\operatorname{arcsec}(x))' = \frac{1}{|x|\sqrt{x^2-1}}}$$

$$\Rightarrow (\operatorname{arccsc}(x))' = \frac{-1}{|x|\sqrt{x^2-1}}$$

Summary:

$$(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos(x))' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan(x))' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot}(x))' = -\frac{1}{1+x^2}$$

$$(\operatorname{arcsec}(x))' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$(\operatorname{arccsc}(x))' = -\frac{1}{|x|\sqrt{x^2-1}}$$

Quiz

1. What is your name?
2. Predict score TCU vs. WVA. American football
(+5 if within 2 pts).
3. Simplify.
   a. $$\frac{x^2p^2 - p^3}{3p^5 + p^3}$$
   b. $\cos(2\theta) + 2\sin^2(\theta)$.
4. If displacement $s(t)$ satisfies
$$s(t) = t^2 - 3t^3;$$
find the velocity equation.